

# IALA GUIDELINE

G1148

## DETERMINATION OF REQUIRED LUMINOUS INTENSITY FOR MARINE SIGNAL LIGHTS

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# DOCUMENT HISTORY

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IALA WORKING PAPER



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## 1 INTRODUCTION

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A suitable luminous intensity is one of the basic features of an AtoN light. The intensity should be sufficient to ensure that the light is visible for navigation as required. It should not be too intense in order to avoid glare.

The purchasing and operating costs are strongly influenced by the luminous intensity of the AtoN light. A 10 cd light can be realised by a low cost and small lantern immediately available from stock of a manufacturer. A sector light of 1 000 000 cd will be expensive, require a special design and will be produced for a single application.

This Guideline should enable the reader to determine the light intensity to provide a good service to the user in a given area, whilst maintaining a balance between performance and cost.

## 2 APPROACH

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There may be several approaches to designing an AtoN light that is suitable to take into account the local conditions around the AtoN. The approach taken in this Guideline attempts to capture most, if not all, the issues surrounding the design of the light.

The approach is broken into two major sections: AtoN light intensity requirement and AtoN light intensity implementation. These two sections deal with two different aspects of ensuring that the AtoN light performs as the user expects.

The AtoN light intensity requirement determines what light intensity is required from the AtoN, given local conditions surrounding the light and the user. This section will result in the light intensity that is used for the officially published nominal range in the List of Lights and nautical charts.

However, there is often additional factors that need to be taken into account that deal with the implementation of the required intensity. These are largely due to design decisions at the AtoN itself that will ultimately impact the intensity produced by the AtoN. This second section will help to ensure that whatever solution is placed at the AtoN, it will produce the required light intensity, and therefore meet the navigational requirements.

The overall process is shown in Figure 1. This figure shows how the different aspects of design come together, and they will be dealt with separately in the sections below. It requires the designer to have determined what the “useable range or area” of the light should be, i.e. over what distances is the light meant to be observed. This will be based on risk analysis of traffic, weather conditions and hazards in the area of interest, and what signal would be required for the user to navigate the area safely.

The process detailed in this Guideline can be considered as an iterative process in order to find the best solution in terms of meeting the navigational requirements and cost.

### 2.1 REPLACING EXISTING LIGHTS

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The approach assumes that a new light is being installed. It can also be used to confirm that an existing light is suitable for the area concerned.

In the event that an existing AtoN light is being renewed, it is possible to determine the design intensity from the published nominal range of the light. However, it is advised that the calculation is completed in full to confirm the correct intensity of the light given the present use and conditions of the area.

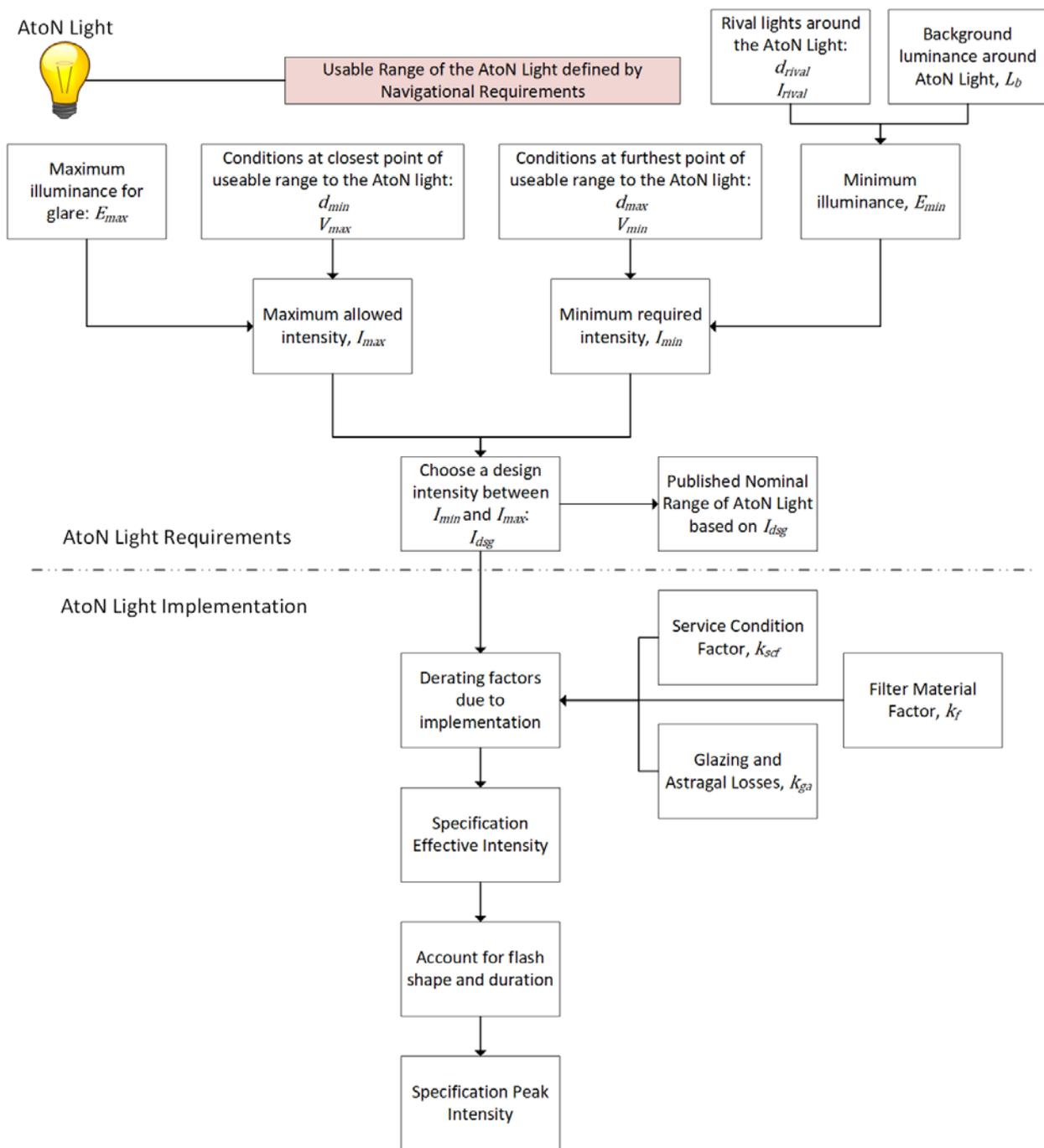


Figure 1 Process of determining the required luminous intensity from the AtoN and light-emitting apparatus.

### 3 DETERMINING ATON LIGHT INTENSITY REQUIREMENT

In this section, the light intensity required from the AtoN that will provide the performance that is expected in a given area will be determined. The light intensity is chosen from a range of intensities between a minimum and a maximum intensity. The process determines what these minimum and maximum intensities are, and then the designer selects an intensity that meets those requirements.

The minimum and maximum intensity selection shall be considered separately.

### 3.1 MAXIMUM ALLOWED INTENSITY

The maximum allowed intensity,  $I_{max}$ , is the maximum intensity that the AtoN light can have. It is determined by three factors that are considered below.

#### 3.1.1 MINIMUM DISTANCE OF OBSERVATION

The minimum distance of observation,  $d_{min}$ , is the distance from the AtoN to the closest point of the useable range or area given in metres. The distance to this area or range can be determined from a map or chart of the area concerned.

#### 3.1.2 MAXIMUM METEOROLOGICAL VISIBILITY

The maximum meteorological visibility,  $V_{max}$ , is the best possible visibility in the local area given in metres. This value, along with the minimum meteorological visibility detailed below, may be difficult to determine without some data to provide an indication of what value it may be. A good source of such information is a local meteorological organisation that may be able to provide visibility data covering the area of interest. Should such data be available, it may be possible to analyse it to determine the level of visibility to a confidence level. For example, the visibility does not exceed 30 km for 95% of the time and this is taken to be the maximum visibility.

In the absence of any useable meteorological data, use  $V_{max} = 20 \text{ M} = 37040 \text{ m}$ .

#### 3.1.3 MAXIMUM ILLUMINANCE FOR GLARE

If the light intensity is too high and/or the observer is very close, then it can cause glare, resulting in uncomfortable viewing for the user. Therefore, it is necessary to determine what is the maximum illuminance at the observer,  $E_{max}$ , that is acceptable given the conditions for the user, given in lx. Two values are given for this, depending on whether there is background luminance, as shown in Table 1.

Background luminance	$E_{max}$ (lx)
None (dark)	0.01
Present	0.1

**Table 1** *Maximum illuminance at the observer*

#### 3.1.4 CALCULATING THE MAXIMUM ALLOWED INTENSITY

In order to calculate the maximum allowed intensity, all the factors above are used in Allard's Law as described in Recommendation R0202 [1]. Thus,

$$I_{max} = E_{max} \cdot d_{min}^2 \cdot 0.05 \frac{d_{min}}{V_{max}}$$

**Equation 1** *Application of Allard's Law for calculating the maximum allowed intensity*

Where:

$I_{max}$  is the maximum allowed intensity (cd)

$E_{max}$  is the maximum allowed illuminance at the observer (lx) (see Section 3.1.3)

$d_{min}$  is the distance to the closest point of the useable range or area to the AtoN light (m) (see Section 3.1.1)

$V_{max}$  is the maximum meteorological visibility in the area of interest (m) (see Section 3.1.2)

## 3.2 MINIMUM REQUIRED INTENSITY

The minimum required intensity,  $I_{min}$ , is the intensity that the AtoN light must provide to ensure that it can be seen throughout the useable range or area in less than optimal conditions. It is determined by a number of factors which are described in detail below.

### 3.2.1 MAXIMUM DISTANCE OF OBSERVATION

The maximum distance of observation,  $d_{max}$ , is the distance from the AtoN to the farthest point of the useable range or area given in metres. This can be determined from a map or chart of the area concerned.

### 3.2.2 MINIMUM METEOROLOGICAL VISIBILITY

The minimum meteorological visibility,  $V_{min}$ , is the worst possible visibility to be considered in the area of interest given in metres. As discussed in Section 3.1.2, it may be difficult to determine without a source of data to confirm typical visibility in poor weather in the area. If such information is available, then as for the maximum meteorological visibility, a statistical approach can be taken to determine this value.

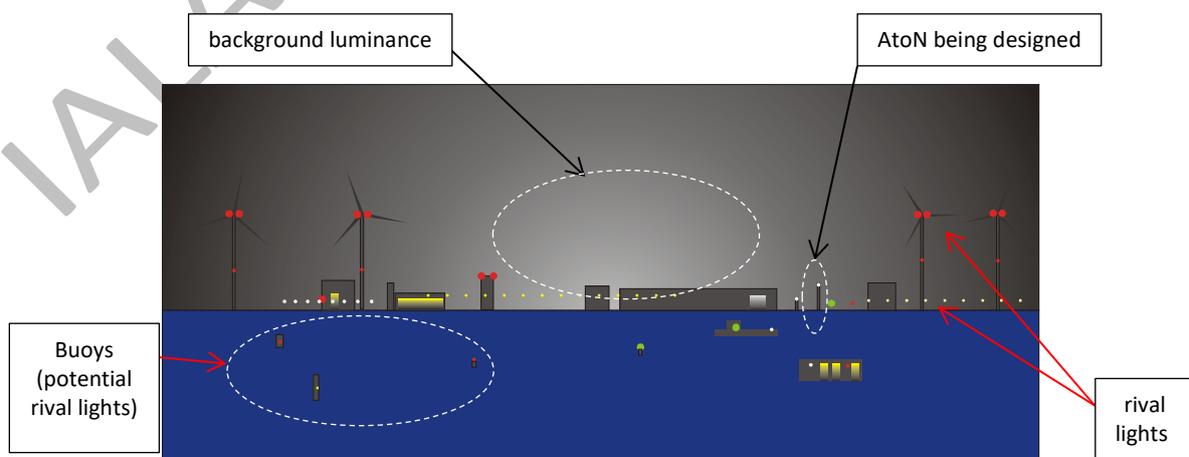
For example, the area of interest may have a visibility of 8 km for 90% of the time. So, for 10% of the time, the visibility is worse than 8 km. However, in deciding the level of performance expected from the AtoN light, the designer may decide that this is acceptable, and therefore use 8 km for the value of  $V_{min}$ .

That said, care must be taken with this approach since it requires a balance between ensuring a good performance from the light in poor weather conditions and the intensity of the light. As the value of  $V_{min}$  lowers, the minimum required intensity increases exponentially, and may end up with an intensity greater than the maximum allowed intensity. This would result in a light that is unnecessarily intense and difficult to observe at close range due to glare.

If no meteorological visibility data is available, then the designer must choose a value that is deemed to be acceptable to the area under consideration. This may require engaging with local users and stakeholders to determine what performance is needed. Note that a higher value of  $V_{min}$  could result in an AtoN light that cannot be seen at the furthest point of the useable range or area under worsening weather conditions.

### 3.2.3 MINIMUM ILLUMINANCE AT THE OBSERVER

The minimum required illuminance at the eye of the observer for identifying the AtoN light,  $E_{min}$ , is dependent on the amount of light that is seen behind and around the AtoN light from the direction of the useable range or area. This background luminance, if sufficiently high, can cause the AtoN light to blend into the background and not be conspicuous. Also, other nearby lights, known as rival lights, can impact the conspicuity of the AtoN light if they are sufficiently intense. Therefore, the AtoN light must have sufficient intensity to contrast against background luminance and be distinct from rival lights. An example of a scenario is shown in Figure 2.



**Figure 2** Example of background luminance and rival lights

Table 2 shows a list of minimum illuminance at the observer required under different conditions. The designer should, if no measurements are available, select the most appropriate condition for the location of the AtoN light being designed. If the background luminance has been measured, then Equation 2 should be used to calculate  $E_{min}$  directly.

Background Condition	Lights	$E_{min}$ ( $\mu\text{lx}$ )
Lights for night-time use		
- no background luminance	all lights except leading lights	0.2
- no background luminance	leading lights	1
- minor background luminance	all lights	2
- substantial background luminance	all lights	20
Lights for daytime use	all lights	1000

**Table 2** *Minimum illuminance to use with different background luminance*

### 3.2.3.1 Known Background Luminance

With a known, possibly measured, background luminance,  $L_b$ , it is possible to calculate the minimum illuminance at the observer by applying Equation 2. This equation has been used historically to determine the required illuminance for observing lights in daytime given the background sky luminance.

$$E_{min} = 0.242 \times 10^{-6} \times (1 + \sqrt{0.4 * L_b})^2$$

**Equation 2** *Minimum illuminance at the observer from background luminance*

Where:

$E_{min}$  is the minimum illuminance at the observer (lx)

$L_b$  is the measured background luminance ( $\text{cd}/\text{m}^2$ )

Equation 2 does not give the same value for  $E_{min}$  when  $L_b = 0$  as Table 2, and therefore, under completely dark background conditions, the value of 0.2  $\mu\text{lx}$  should be used (or 1  $\mu\text{lx}$  for leading lights).

Table 3 shows the required illuminance of the AtoN light under different daytime conditions. If typical meteorological conditions can be cited for the location of the AtoN, then this table can be used to ensure that the light intensity is sufficient to be seen in the useable range or area during those conditions.

Meteorological Condition	Luminance, $L_b$ ( $\text{cd}/\text{m}^2$ )	Required Illuminance, $E_{min}$ ( $\mu\text{lx}$ )
Very dark overcast sky	100	13
Dark overcast sky	200	24
Ordinary overcast sky	1000	107
Bright overcast sky or clear sky away from the direction of the sun	5000	506
Bright cloud or clear sky close to the direction of the sun	10000	1000
Very bright cloud	20000	1980
Glaring cloud	50000	4910

**Table 3** *Required illuminance under different daytime conditions*

### 3.2.3.2 Rival Lights

Rival lights are point sources of light in the area of observation around the AtoN light. If an excessive number of rival lights exist, or they are of higher intensity than the AtoN light, then it can make the AtoN light difficult to discern and less conspicuous. Such an example is shown in Figure 2. Therefore, the minimum illuminance produced by the AtoN light should be equal to or greater than the illuminance produced by the rival light.

Allard's Law is applied to establish the illuminance produced by a rival light. Thus,

$$E_{rival} = \frac{I_{rival}}{d_{rival}^2 \cdot 0.05 \frac{d_{rival}}{V_{min}}}$$

**Equation 3** *Illuminance at the observer caused by rival lights*

Where:

$E_{rival}$  is the illuminance at the observer produced by the rival light (lx)

$I_{rival}$  is the intensity of the rival light (cd)

$d_{rival}$  is the distance of farthest point of the useable range or area to the rival light (m)

$V_{min}$  is the minimum meteorological visibility in the area of interest (m) (see Section 3.2.2)

Equation 3 must be applied to all major rival light sources in the vicinity of the AtoN light to ensure that the most prominent light source is captured. It will be necessary to determine the distance from the rival light to the furthest point of the useable range or area, and this should be possible using a map of the area of interest. The intensity of the rival light is more difficult to establish. To assist in this calculation, Table 4 contains some typical intensities for a number of potential rival light sources. An initial field-of-view to consider rival lights should be 1° either side of the AtoN light when viewing the AtoN from the furthest point in useable range or area of the AtoN. The field of view can be expanded if a particularly intense rival light has been identified outside of the initial field-of-view.

Once  $E_{rival}$  has been calculated for all rival lights, the highest value is selected and compared to  $E_{min}$  determined above. If  $E_{rival}$  is greater than  $E_{min}$ , then  $E_{min}$  should be revised to be equal to or greater than  $E_{rival}$ . The greater the value of  $E_{min}$  above  $E_{rival}$ , the more conspicuous the AtoN light will be in comparison to the rival light.

Purpose	Type	Luminous intensity (cd)	Characteristics
Aeronautical obstacle lights	Low-intensity, Type A, red, fixed	10	omnidirectional
	Low-intensity, Type B, red, fixed	30	omnidirectional
	Medium intensity, Type A, white, flashing	2000	omnidirectional
	Medium intensity, Type B, red, flashing	2000	omnidirectional
Road traffic lights	Green, Red, Yellow	25 to 200	pencil beam
Vessel Navigation Lights (Length <12 m)	White	4.3	Sectored
	Red, Green	0.9	Sectored
Vessel Navigation Lights (Length between 12 and 50 m)	White	12 to 52	Sectored
	Red, Green	4.3	Sectored
Vessel Navigation Lights (Length >50 m)	White	94	Sectored
	Red, Green	12	Sectored

**Table 4** *Typical rival lights intensities*

### 3.2.4 CALCULATING THE MINIMUM REQUIRED INTENSITY

In order to calculate the minimum required intensity, all the factors above are used in Allard's Law as described in Recommendation R0202 [1]. Thus,

$$I_{min} = E_{min} \cdot d_{max}^2 \cdot 0.05 \frac{d_{max}}{V_{min}}$$

Equation 4    Application of Allard's Law for calculating the minimum required intensity

Where:

$I_{min}$  is the minimum required intensity (cd)

$E_{min}$  is the minimum illuminance at the observer (lx) (see Section 3.2.3)

$d_{max}$  is the distance of farthest point of the useable range to the AtoN light (m) (see Section 3.2.1)

$V_{min}$  is the minimum meteorological visibility in the area of interest (m) (see Section 3.2.2)

### 3.3 SELECTING A DESIGN INTENSITY

The design intensity,  $I_{dsg}$ , is an intensity chosen by the designer to sit between the minimum and maximum required intensities calculated above. Thus,

$$I_{min} \leq I_{dsg} \leq I_{max}$$

In many cases, the design intensity will be made equal to the minimum required intensity. However, there may be operational reasons that a designer will want to choose a higher design intensity (e.g. a desire to be more conspicuous than usual in a given area), in which case the designer can choose a value higher than the minimum (but not greater than the maximum allowed intensity).

The design intensity will be used for subsequent calculations.

#### 3.3.1 NOMINAL RANGE FOR PUBLISHING

The design intensity is the intensity of light exhibited from the AtoN. As such, it is this value which the published nominal range should be based upon. The calculation of the nominal range is based on Allard's Law using pre-defined conditions, resulting in Equation 5.

$$I_{dsg} = E_{nominal} \cdot d_{nominal}^2 \cdot 0.05 \frac{d_{nominal}}{18520}$$

Equation 5    Application of Allard's Law for calculating the nominal range

Where:

$I_{dsg}$  is the design intensity (cd)

$E_{nominal}$  is the illuminance at the observer under nominal conditions (lx).

For daytime,  $E_{nominal} = 1000 \mu\text{lx}$ .

For night-time,  $E_{nominal} = 0.2 \mu\text{lx}$ .

$d_{nominal}$  is the nominal range of the AtoN light (m)

The nominal range is given in metres, and for publishing it in nautical publications will need to be converted to nautical miles by dividing the result by 1852 (e.g. 5000 m = 2.7 M). The nominal range, by definition, assumes a meteorological visibility of 10 M (= 18520 m). Note that the illuminance at the observer only has two values in this calculation: one for night-time and one for daytime. These are the only values that should be used for calculating nominal range, regardless of what values were used in calculating the design intensity.

Unfortunately, it is not possible to calculate  $d_{nominal}$  directly by rearranging Equation 5. It must be determined by other means. A common method is to use a reverse lookup table where the intensity for different ranges is calculated. It is also possible to use an iterative process where the range is updated between each calculation to get the value of the intensity closer to the one required (such as the method described in ANNEX A). Each method have their pros and cons, and it is up to the reader to determine the best solution for their application. A quick spreadsheet function can be used, as shown below, to calculate the nominal range using the design intensity in cell B1 (in cd) and the value for  $E_{nominal}$  in cell C1 (in lx):

```
=INDEX(ROW($A$1:$A$4000)*0.01,MATCH($B$1,$C$1*((ROW($A$1:$A$4000)*18.52)^2)*0.05^(-ROW($A$1:$A$4000)*18.52/18520),1))
```

This formula will give the nominal range with a resolution of 0.01 M up to 40 M, but it *must* be entered as an array formula, i.e. press CTRL-SHIFT-ENTER instead of the usual ENTER to enter the formula into the cell. Although the range \$A\$1:\$A\$4000 is referred to in the function, the contents of those cells are not used. It may be necessary to modify the formula to enable it to work with the spreadsheet locale settings (e.g. decimal separator).

## 4 IMPLEMENTING ATON LIGHT INTENSITY REQUIREMENT

In the previous Chapter, the process of determining the design intensity was discussed. This intensity is the light output from the AtoN, and therefore is the value that the published nominal range is based upon.

However, there are a number of factors that impact the light output of the light-emitting apparatus and the output from the AtoN. These factors need to be taken into account so that the equipment of the correct specification can be determined. Failure to do this may cause the AtoN light to fail to meet its published performance, and it will be a degraded service.

### 4.1 DE-RATING FACTORS DUE TO IMPLEMENTATION

In this section, we will apply several de-rating factors to the design intensity to determine what intensity the light-emitting apparatus should be performing at to ensure the design intensity leaving the AtoN in the worst case scenario. This will be the specification intensity, so called because it should be the intensity specified by authorities when procuring light-emitting apparatus for the AtoN light under consideration.

The designer should consider each of the factors given below, and determine what value should apply for each one. These de-rating factors will then be applied to the design intensity to determine the specification intensity.

#### 4.1.1 SERVICE CONDITION FACTOR

The service condition factor is a means of allowing for the light output from an AtoN to degrade and continue to perform as published. It should capture all effects that are under the control of the operating authority through maintenance. The effects of dirt, dust, optical degradation of materials and light source degradation should all be captured in this factor.

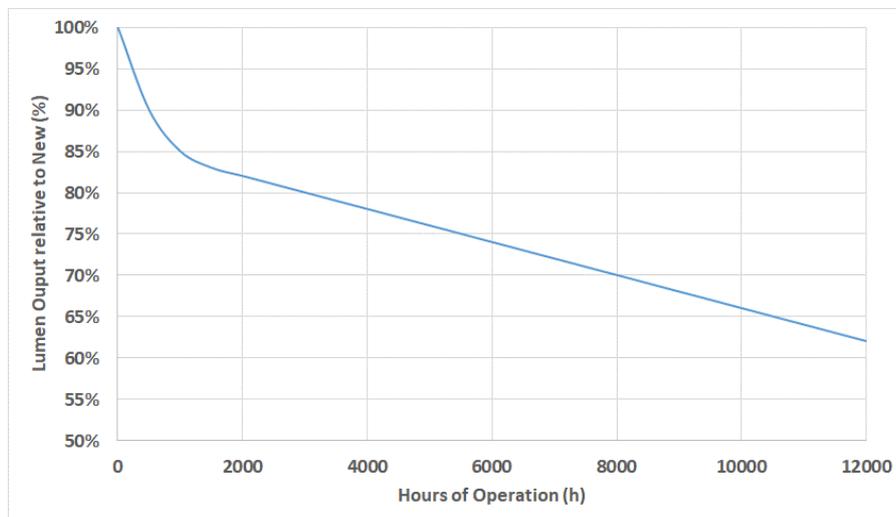
It should be recognised that the value for the service condition factor is highly dependent on the maintenance regime employed at the AtoN. For example, if a lighthouse is cleaned on a weekly basis, the service condition factor may be much less than a lighthouse that is cleaned on an annual basis because the build-up of dirt and dust will not be as great.

It may be useful to consult ISO/CIE Technical Specification 22012 regarding Maintenance Factor Determination, which covers the subject equivalent to Service Condition Factor for general lighting calculations.

##### 4.1.1.1 Light Source Degradation

All light sources degrade with time, but some light sources degrade more quickly than others. Manufacturers normally provide some form of lumen depreciation information in the datasheets of their lamps. Such information is invaluable to identify the limits of the light sources used in the AtoN.

An example of the lumen depreciation of a light source with hours of operation is shown in Figure 3. It shows that, for this particular light source, there is a quick drop in lumen output early on in its lifetime, followed by a more gradual drop in output as time goes on. If the light source is going to be operating normally before being changed, then this lumen depreciation must be accounted for. If the light source is changed after 4000 hours of operation, then around 22% of lumen depreciation must be accounted for. If the light source is changed after 10000 hours of operation, then 34% of lumen depreciation must be accounted for.



**Figure 3** *Example of a lumen depreciation of a light source with hours of operation*

Using such information from manufacturers, it is possible to determine the value for the light source degradation,  $l_l$ . This would be the amount of lumen depreciation exhibited by the light source at the end of its service life. In the example above, if the light source is changed after 4000 hours, then  $l_l = 0.22$  (= 22 %).

As a general rule, LED technology tends to exhibit much less lumen depreciation than other types of light sources for the same hours of operation. However, since LEDs have a longer service life than other light sources, the value of  $l_l$  for LEDs is not 0, and can be as high as 0.2 for 25000 hours of service. It is best to confirm the lumen depreciation information with the manufacturer of the LED to identify a suitable value for  $l_l$ . Values of 0.05 to 0.1 (equal to 5 % to 10 % loss) may be typical. It should be recognised that this value is unlikely to be precise due to the variable operational conditions of light sources such as LED.

In general terms,

$$l_l = \frac{\Phi_n - \Phi_s}{\Phi_n}$$

**Equation 6** *Calculation of lumen degradation within the light source*

Where  $l_l$  is the loss due to lumen depreciation during the service life of the light source.

$\Phi_n$  is the total luminous flux of the light source in new condition (lm).

$\Phi_s$  is the total luminous flux of the light source at the end of its service life (lm).

#### 4.1.1.2 Light Source Failure

Some light sources are formed of several individual smaller light sources operating together to form a single light source. A typical example of this are integrated lanterns where several LEDs arranged in a ring operate together to effectively form a single light source. The actual loss caused by a light source failure is dependent on the design of the optical and electrical system of the light source.

During the service life of a multi-source light source, there will be a certain probability of one of the individual light sources (or drivers) to fail. This loss,  $l_f$ , accounts for the drop of total lumen output of the light source due to the individual failure. Thus,

$$l_f = \frac{\Phi_n - \Phi_f}{\Phi_n}$$

**Equation 7** *Calculation of lumen degradation due to failure within the light source*

Where  $l_f$  is the loss due to failure within the light source.

$\Phi_n$  is the total luminous flux of the light source in normal operating condition (lm).

$\Phi_f$  is the total luminous flux of the light source in failure operating condition (lm).

It may be easier to determine the value based on the 10<sup>th</sup>-percentile azimuth intensity value (see IALA Recommendation E200-3) of the multi-source light source, such that:

$$l_f = \frac{I_n - I_f}{I_n}$$

**Equation 8** *Calculation of lumen degradation due to failure within the light source using intensity*

Where  $l_f$  is the loss due to failure within the light source.

$I_n$  is the 10<sup>th</sup>-percentile azimuth intensity of the light source in normal operating condition (cd).

$I_f$  is the 10<sup>th</sup>-percentile azimuth intensity of the light source in failure operating condition (cd).

If a failure of a light source is automatically fixed by the AtoN control system by replacing it with an identical light source, then  $l_f = 0$ . Examples of such systems are the use of lampchangers or having an identical standby integrated lantern operating on detection of a failure of the main integrated lantern. Also, this factor can be ignored if the maintenance regime means that on a failure of a light source, the light source is replaced.

#### 4.1.1.3 Optical Path Degradation

This, perhaps, is the most difficult type of degradation to quantify since it is rare for any form of relevant data to exist for marine aids-to-navigation. There will be variation from one AtoN to another, and quite often variation between different directions on the same AtoN due to prevailing weather conditions or a tendency for wildlife to gather at a certain location on the AtoN. Examples of such degradation are shown in Figure 4.



**Figure 4** *Examples of bird fouling accumulating on lighthouse glazing (Source: Trinity House and Danish Maritime Authority)*

Some attempts have been made to quantify the optical path degradation in the UK and Ireland, and the results have been surprising. Using a measurement process where the intensity of the beam created by lighthouses are measured in the field, it has been possible to determine the impact of cleaning on the glazing and optics of the lighthouse.

Measurements were carried out at three different lighthouses, and the losses due to dust/dirt accumulating on the traditional optics were measured to be 45 %, 0 % and 32 %. Similarly, the losses due to dust/dirt/salt on the glazing at two of the lighthouses were 12 % and 30 %. Unfortunately, it has not been possible to determine over what time period the accumulation of dirt and dust occurred in these cases.

Optical path degradation should also consider the degradation in the material of the optic and glazing. It should include UV degradation and surface damage of the material. Degradation of plastic components can be severe if they are not UV-stabilised. An example of such degradation is shown in Figure 5.



**Figure 5** *Example of different degrees of degradation of glazing panels made from plastic (diamond) and glass (triangular). Source: MSM*

In the vast majority of cases, the best that can be achieved in quantifying the optical path degradation is to estimate it based on the experience and knowledge of the environment that the AtoN is in and how regular the maintenance teams can visit to clean the AtoN. Using this information, the value of optical path degradation  $l_d$  is set to the ratio between the amount of light lost due to dirt and dust and the amount of light in clean conditions.

In case of no information being available, a value of  $l_d = 0.25$  may be acceptable, which is equal to a maximum of 25 % loss of light due to dirt and dust.

#### 4.1.1.4 Determining the Service Condition Factor

The service condition factor brings together all the degradation elements from above into a single factor to be applied to the design intensity. It is calculated by:

$$k_{scf} = (1 - l_l)(1 - l_f)(1 - l_d)$$

**Equation 9** *Calculating the Service Condition Factor*

Where  $k_{scf}$  is the Service Condition Factor.

$l_l$  is the lumen depreciation of the light source at the end of its service life (see Section 4.1.1.1).

$l_f$  is the lumen depreciation due to a failure during the light source service life (see Section 4.1.1.2).

$l_d$  is the lumen depreciation due to optical path degradation between maintenance visits (see Section 4.1.1.3).

#### 4.1.2 ASTRAGAL AND GLAZING LOSS FACTOR

When the light-emitting apparatus is installed inside a lantern room or some structure, losses can be experienced due to:

- light absorption, reflection and scattering in the glazing material, and
- obscuration caused by solid structures (e.g. astragals).

The losses experienced by these means are fixed – they are inherent in the design of the AtoN. Different designs and materials will experience different losses, and therefore must be considered on a per design basis.

##### 4.1.2.1 Glazing Material Loss

The type of glazing used in an AtoN can have an impact on the amount of light transmitting through it. The amount of this loss is purely subject to the type of material that is used – the issue of cleanliness is dealt with under service condition factor in Section 4.1.1. The loss due to glazing is given as the factor,  $k_a$ .

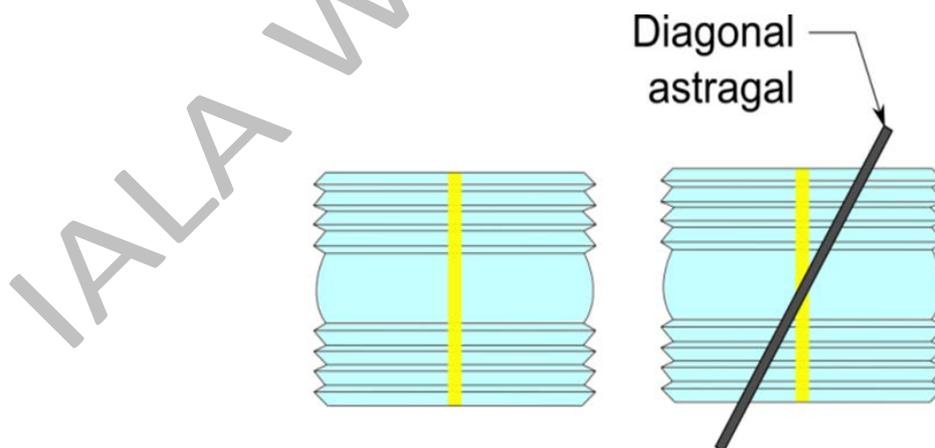
A typical value of loss through glass is approximately 10 %, i.e.  $l_g = 0.1$ . However, acrylic glazing substitute can have a loss as low as 2 % ( $l_g = 0.02$ ). A value for the transmittance (and therefore the loss) can be found from datasheets, consulting with the manufacturer or by measurement. In the event that no such information is available, assume 10 % as the loss through glazing.

If no glazing material is used between the light-emitting apparatus and outside of the AtoN, then  $l_g = 0$ .

##### 4.1.2.2 Astragal Obscuration Loss

Astragals, or any other solid structure, between the light emitting apparatus and the user will cause obscuration, and reduce the light intensity in certain directions. In the most extreme cases, it is possible to completely obscure the light (e.g. using vertical astragals with fixed optics). The amount of obscuration depends entirely on the size of light projected outwards and the nature of the structure surrounding it. For this reason it is possible to change the impact of astragals by modifying the light distribution from the light-emitting apparatus.

It is possible to calculate the effects of astragals on the light intensity if the size of the projected light source and the structures surrounding it are known. However, it is accepted that it is a difficult calculation. It may be possible to estimate the impact of astragals by simplifying the system to a single situation, and assume that the level of obscuration is constant in the direction of the useable range or area.



**Figure 6** *Demonstration of astragal obscuration on the total light output from a fixed optic*

Take the example shown in Figure 6. This figure shows the situation of a fixed optic without astragal obscuration and with astragal obscuration. The loss,  $l_a$ , represents the difference in the observed lit area between these two situations. The lit area obscured by the astragal can be determined using Equation 10.

$$A_o = \frac{n_c w_a w_l}{\sin \theta}$$

Equation 10 Obscured area of a fixed optic light due to astragals

Where:

$A_o$  is the area of obscuration ( $m^2$ ).

$n_c$  is the number of astragals crossing the light source from a given direction.

$w_a$  is the width of the astragal (m).

$w_l$  is the width of the light source (m).

$\theta$  is the angle of the astragal from vertical.

The obscuration loss due to astragals is

$$l_a = \frac{A_o - A_u}{A_u}$$

Equation 11 Estimation of the loss due to astragal obscuration

Where:

$l_a$  is the loss due to astragals.

$A_o$  is the total area of obscuration due to astragals in  $m^2$ .

$A_u$  is the total unobscured area of the lit area in  $m^2$ .

If the light from the light emitting apparatus is unobstructed by astragals (or other structures) in the direction of the useable range or area, then  $l_a = 0$ .

In the event where information on the astragals is not available or too complex to model, then it may be necessary to estimate the value of  $l_a$ .

#### 4.1.2.3 Determining the Glazing and Astragal Factor

The glazing and astragal factor,  $k_{ga}$  brings the two losses described above into a single factor that determines the overall reduction in the light intensity due to these effects. Thus,

$$k_{ga} = (1 - l_g)(1 - l_a)$$

Equation 12 Glazing and Astragal Factor

Where:

$k_{ga}$  is the glazing and astragal factor.

$l_g$  is the glazing material loss (see Section 4.1.2.1).

$l_a$  is the astragal obscuration loss (see Section 4.1.2.2).

In the past, a value of  $l_a = 0.06$  was used combined with a glazing loss of  $l_g = 0.1$  to produce a glazing/astragal factor of 0.85. This was used for traditional optics that are large relative to the size of the astragals. The modern practice of using smaller optics may have an effect of increasing the value of  $l_a$ , but it is dependent on the size of the light source projected from the AtoN and the location and dimension of the astragals relative to the projected light.

#### 4.1.3 FILTER MATERIAL FACTOR

Many lighthouses have sectors created by the use of colour filters in order for a light colour to be different in a given direction. By their very nature, filters attenuate the total light as it absorbs light that does not fit into its passband. For example, a red filter will absorb blue, green and yellow. This means that any light energy in those

wavelengths will be lost, causing the intensity to drop. If filters are used, then the drop in intensity due to their use needs to be taken into account.

The factor depends on the spectrum of the light and the wavelength dependant transmittance of the filter material. Both quantities can either be measured to derive an overall transmittance, or the overall transmittance can be measured directly with a photometer. Typical values for red, green and yellow filters with different light sources are shown in Table 5. These values are only for guidance, and the transmittance for the filters used should be measured using the light source to be used in the AtoN. The designer should also ensure that the colour produced complies with IALA Recommendation R0201. More information on the use of filter material in sector lights can be found in IALA Guideline G1041.

	Value of $k_f$		
	Red Filter	Green Filter	Yellow Filter
<b>White LED</b>	0.05 – 0.15	0.15 – 0.20	0.45 – 0.50
<b>Tungsten Halogen</b>	0.08 – 0.25	0.08 – 0.25	0.50 – 0.70

**Table 5** *Typical filter material factors for LED and Tungsten Halogen lamps*

Note that if no additional colour filters to those inside the light-emitting apparatus are used, then this factor can be ignored (i.e.  $k_f = 1$ ) because it will be captured in the specification intensities below.

## 4.2 SPECIFICATION EFFECTIVE INTENSITY

The Specification Effective Intensity,  $I_{se}$ , is the minimum effective intensity of the light that will meet the Design Intensity after taking the de-rating factors into account. Note that this is the *effective* intensity – the intensity of a flashing light that is perceived. Recall that the design intensity is the intensity that the user observes regardless of the flash duration and profile. As such, the design intensity is also the effective intensity.

Thus,

$$I_{se} = \frac{I_{dsg}}{k_{scf} \cdot k_{ga} \cdot k_f}$$

**Equation 13** *Calculation of the Specification Effective Intensity*

Where:

$I_{se}$  is the Specification Effective Intensity (cd).

$I_{dsg}$  is the Design Intensity (cd) (see Section 3.3)

$k_{scf}$  is the service condition factor (see Section 4.1.1)

$k_{ga}$  is the glazing and astragal loss (see Section 4.1.2)

$k_f$  is the filter material factor (see Section 4.1.3)

The Specification Effective Intensity is fixed only by the parameters given above – it does not take into account the flash character. This could be advantageous since knowledge of the flash shape is not required at the point of procurement – only the Specification Effective Intensity and the shortest flash duration within the character(s) is needed. With these two parameters, it would be possible to determine whether a light source can meet the requirements of the aids to navigation.

It should be noted that as discussed in Section 4.1.2.2, the amount of obscuration is dependent on the nature of the light projection out of the light emitting apparatus. There may be room to adjust  $k_{ga}$  if a favourable light distribution is obtained from a particular design approach, thus allowing some adjustment in the Specification

Effective Intensity. Care should be taken to make such adjustments since a too favourable value would result in a light emitting apparatus not providing the Design Intensity required.

### 4.3 SPECIFICATION PEAK INTENSITY

There is a difference between the effective and peak intensities due to the human visual response to flashing lights, which means that the peak intensity of a light is not perceived immediately. More information on effective intensity can be found in Recommendation R0204 and Guideline G1135. The peak intensity is calculated by applying a model, known as the Modified Allard Method, which takes into account the human visual response to a light with a known flash profile. If the flash profile of the light-emitting apparatus is known, then it is possible to determine a factor,  $k_e$ , to convert the Specification Effective Intensity to the Specification Peak Intensity,  $I_{sp}$ . Thus,

$$I_{sp} = \frac{I_{se}}{k_e}$$

*Equation 14 Specification Peak Intensity*

Where:

$I_{sp}$  is the Specification Peak Intensity (cd)

$I_{se}$  is the Specification Effective Intensity (cd) (see Section 4.2)

$k_e$  is the factor taking into account the flash profile determined by the method given in IALA Recommendation R0204.

As described in IALA Guideline G1135, for a perfectly rectangular flash profile, as in case of unmodified flashes of LED lanterns, it is possible to apply the Blondel-Rey equation to calculate,  $k_e$ . Thus,

$$k_e = \frac{T}{T + a}$$

*Equation 15 Application of Blondel-Rey equation for rectangular flash profiles*

Where:

$k_e$  is the factor taking into account the flash profile.

$T$  is the shortest flash duration in a flash character (s).

$a$  is the visual constant (s). It is equal to 0.1 s, except for blue signal light at night where  $a = 0.2$  s.

For non-rectangular flashes, as in case of incandescent lamps, rotating lanterns and LED lanterns with modified flash shape, the full Modified Allard Method will have to be applied in order to determine  $k_e$ . See IALA Guideline G1135 for more details.

For steady-burning lights,  $k_e = 1$ , meaning that the Specification Peak Intensity is the same as the Specification Effective Intensity.

## 5 EXAMPLES

### 5.1 EXAMPLE 1 – FINDING THE INTENSITY BASED ON THE RANGE

A lighthouse is being refitted with a light to serve an area from 1 M to 8 M from the lighthouse. It is necessary for the light to be seen for at least 50 % of the time in prevailing weather conditions. LED lantern is used with rectangular flash profile and flash character is Fl W (2) 10 s, with a flash duration of 0.5 seconds. The light will only be exhibited during the night. The lantern is located in the lantern room causing 15% loss of intensity due to glazing and astragals. There are no colour filters used. The lighthouse is located in a place where air is relatively clean and birds or salt spray from waves cause 25% of loss of intensity between maintenance visits. The peak intensity for technical specification for procurement of the lantern must be determined.

#### 5.1.1 MINIMUM DISTANCE OF OBSERVATION

In this example, the minimum distance of observation is 1 M. So,  $d_{min} = 1852$  m.

#### 5.1.2 MAXIMUM METEOROLOGICAL VISIBILITY

Using the local meteorological office data, the best meteorological visibility is considered to be 20 M. So,  $V_{max} = 37040$  m.

#### 5.1.3 MAXIMUM ILLUMINANCE FOR GLARE

On inspection of the background of the lighthouse from the direction of the area of utilisation, it was found that there is no background luminance. Therefore,  $E_{max} = 0.01$  lx.

#### 5.1.4 CALCULATING THE MAXIMUM ALLOWED INTENSITY

By applying Equation 1, we find that the maximum allowed intensity is:

$$\begin{aligned} I_{max} &= E_{max} \cdot d_{min}^2 \cdot 0.05 \frac{d_{min}}{V_{max}} \\ I_{max} &= 0.01 \cdot 1852^2 \cdot 0.05 \frac{1852}{37040} \\ I_{max} &= 39841 \text{ cd} \end{aligned}$$

Therefore, the maximum intensity of the light emitted from the lighthouse can be 39841 cd.

#### 5.1.5 MAXIMUM DISTANCE OF OBSERVATION

The maximum distance of observation is 8 M. So,  $d_{max} = 14816$  m.

#### 5.1.6 MINIMUM METEOROLOGICAL VISIBILITY

There is a requirement for the light to be seen in the area of utilisation for 50 % of the time. Using data from the local meteorological office, the visibility is at least 5 M for 50 % of the time. So,  $V_{min} = 9260$  m.

#### 5.1.7 MINIMUM ILLUMINANCE AT THE OBSERVER

The light will only be exhibited during the night, and there is no background luminance. Therefore, according to Table 2,  $E_{min} = 0.2$   $\mu$ lx.

Also, there are no rival lights in the area, and therefore no adjustment to  $E_{min}$  is required.

#### 5.1.8 CALCULATING THE MINIMUM REQUIRED INTENSITY

By applying Equation 4, we find that the minimum intensity is:

$$\begin{aligned} I_{min} &= E_{min} \cdot d_{max}^2 \cdot 0.05 \frac{d_{max}}{V_{min}} \\ I_{min} &= 0.0000002 \cdot 14816^2 \cdot 0.05 \frac{14816}{9260} \\ I_{min} &= 5298 \text{ cd} \end{aligned}$$

Therefore, the minimum required intensity for the light to be seen at the furthest point in the area of utilisation under the adverse weather conditions prescribed in the requirements is 5298 cd.

### 5.1.9 SELECTING A DESIGN INTENSITY

We wish to save as much power as possible, and since there is no need to be extra conspicuous, it is decided that the design intensity,  $I_{dsg} = I_{min} = 5298$  cd.

#### 5.1.9.1 Nominal Range for Publishing

Now that the design intensity has been selected, the nominal range for publishing can be calculated. Using Equation 5 or the table in Recommendation R0202, we find that 5298 cd has a nominal range of 13 M to the nearest whole nautical mile.

#### 5.1.10 CALCULATING THE SPECIFICATION EFFECTIVE INTENSITY

To find specification peak intensity, applicable derating factors have to be applied to the design intensity.

The lantern is located in the lantern room causing losses of intensity of 15%. Therefore, the derating factor for glazing and astragals of the lantern room,  $k_{ga}$ , is 0.85.

As no filter is used, the derating factor for filter,  $k_f$ , is 1.

The service conditions cause losses of intensity between maintenance visits of 25%. Therefore, the derating factor for service conditions,  $k_{scf}$ , is 0.75.

By applying Equation 4, we find that the specification effective intensity is:

$$I_{se} = \frac{I_{dsg}}{k_{scf} \cdot k_{ga} \cdot k_f}$$
$$I_{se} = \frac{5298}{0.75 \cdot 0.85 \cdot 1}$$
$$I_{se} = 8310 \text{ cd}$$

So, the required specification effective intensity is 8310 cd.

#### 5.1.11 CALCULATING THE SPECIFICATION PEAK INTENSITY

Required peak intensity of flashing light is calculated with the Equation 4:

$$I_{sp} = \frac{I_{se}}{k_e}$$

The flash shape is rectangular, and therefore  $k_e$  is calculated with Equation 45:

$$k_e = \frac{T}{T + a}$$
$$k_e = \frac{0.5}{0.5 + 0.1}$$
$$k_e = 0.83$$
$$I_{sp} = \frac{8310}{0.83}$$
$$I_{sp} = 9972 \text{ cd}$$

So, the required peak intensity of the flashing light specified in procurement documents has to be 9972 or ~10000 cd.

## 5.2 EXAMPLE 2 – FINDING THE RANGE BASED ON THE INTENSITY

A port authority wishes to maximise the distance between their channel markers, whilst ensuring that the lights can still be seen from an adjacent marker.

The authority's standard buoy lantern has the peak intensity of 15 cd. The light is only used at night time.

LED lantern is used with rectangular flash profile and the flash character is Fl G 5 s, with a flash duration of 0.4 seconds.

Visibility in the area is at least 4 M for 90% of the time. As the port authority is interested in the range of lights in the worst case scenario, this is used as the minimum visibility.

It has been found that between maintenance visits, the light intensity falls by 30% due to build-up of bird fouling and dirt.

Luminance of the background seen when navigating in the channel has been measured to be 0.08 cd/m<sup>2</sup>.

The range of the lanterns for determining the maximum distance between the adjacent markers must be determined.

### 5.2.1 CALCULATING EFFECTIVE INTENSITY

As the known intensity of the lantern is peak intensity the effective intensity of a flash has to be determined. The formula for calculating the effective intensity of flashing light from the peak intensity is derived from the **Error! Reference source not found.**

$$I_{sp} = \frac{I_{se}}{k_e}$$

$$I_{se} = I_{sp} \cdot k_e$$

The flash shape of the LED lights is rectangular, and therefore  $k_e$  is calculated with **Error! Reference source not found.** starting from the known peak intensity:

$$I_{sp} = 15 \text{ cd}$$

$$k_e = \frac{T}{T + a}$$

$$k_e = \frac{0.4}{0.4 + 0.1}$$

$$k_e = 0.8$$

Thus,

$$I_{se} = 15 \cdot 0.8$$

$$I_{se} = 12 \text{ cd}$$

### 5.2.2 GETTING DESIGN INTENSITY BY APPLYING SERVICE CONDITIONS FACTOR

To obtain the intensity of the light in the worst case, i.e. at the end of the service life and maintenance cycle of the lantern, the intensity has to be derated by the service conditions factor  $k_{scf} = 0.7$  due to losses due to optical path degradation  $l_a = 0.3$ . As the lantern is not in a lantern room, colour filters are not used and it is assumed that the lantern will be replaced before significant lumen depreciation of the LED has taken place, there are no other derating factors. Thus, using **Error! Reference source not found.**, we can calculate the design intensity:

$$I_{se} = \frac{I_{dsg}}{k_{scf} \cdot k_{ga} \cdot k_f}$$

$$I_{dsg} = I_{se} \cdot k_{scf}$$

$$I_{dsg} = 12 \cdot 0.7$$

$$I_{dsg} = 8.4 \text{ cd}$$

### 5.2.3 CALCULATING THE RANGE CONSIDERING BACKGROUND LUMINANCE AND VISIBILITY

To calculate the range, the minimum required illuminance  $E_{min}$  in the given conditions (background luminance  $L_b = 0.08 \text{ cd/m}^2$ ) is needed. Minimum required illuminance is calculated with the Equation 2.

$$E_{min} = 0.242 \times 10^{-6} \times (1 + \sqrt{0.4 * L_b})^2$$

$$E_{min} = 0.242 \times 10^{-6} \times (1 + \sqrt{0.4 * 0.08})^2$$

$$E_{min} = 0.336 \text{ } \mu\text{l x}$$

Now all the data for calculation of the range is available. The range can be calculated using based method on Allard's Law (ANNEX A), with the values of  $I_{dsg} = 8.4 \text{ cd}$ ,  $E_{min} = 0.336 \text{ } \mu\text{l x}$  and  $V_{min} = 7408 \text{ m}$ . Other methods of determining the range are available such as the ones described in Section 3.3.1.

$$I_{dsg} = E_{min} \cdot d_{max}^2 \cdot 0.05 \frac{d_{max}}{V_{max}}$$

For this task, it was decided to use a modified version of the spreadsheet function mentioned in Chapter 3.3.1 to allow for a different meteorological visibility. The value in meters corresponding to the visibility of 10 M in the function is replaced with the value in meters corresponding to 4 M, i.e. the figure 18520 has to be replaced with 7408. It is found that the value of  $d_{max} = 2824 \text{ m}$ , or 1.52 M. This is the luminous range of the light in the given conditions.

As described in the IALA Guideline 1078 on the Use of Aids to Navigation in the Design of Fairways, before reaching a marker, the next channel marker has to be seen already. Therefore, the distance between the adjacent markers has to be less than this maximum range of the lights under the given conditions. Optimal distance between the channel markers depend also on other factors, but this calculation gives good guidance for the maximum separation between markers. Further details on that can be found in IALA Guideline 1078.

## ANNEX A    CALCULATION OF LUMINOUS RANGE FROM INTENSITY

The equation for Allard's Law cannot be rearranged to calculate the luminous range from intensity directly. Therefore, the calculation of the distance requires a numerical approximation, which can be done by applying an iterative process as shown below.

The general form of Allard's Law from IALA Recommendation R0202 is:

$$I = d^2 \cdot E \cdot 0.05^{-\frac{d}{V}}$$

Equation 16    General form of Allard's Law

Where:

$I$  is the intensity of the light (cd)

$d$  is the distance from the light to the observer, which is the luminous range to be found (m)

$E$  is the illuminance at the observer (lx)

$V$  is the meteorological visibility (m)

### **A.1**    **NEWTON-RAPHSON METHOD**

The distance  $d$  can be determined from Allard's law by applying the Newton-Raphson method. This method is used to numerically determine the roots of a mathematical function by starting with an initial value, and refining the value until the difference between iterations become sufficiently small. In this case, it is the value of  $d$  that is changed.

A function  $f$  is defined to be:

$$f(d) = d^2 \cdot E \cdot 0.05^{-\frac{d}{V}} - I$$

Equation 17    Function  $f$

In this case, the distance  $d$  is the root of the function  $f(d)$  such that  $f(d) = 0$ .

For the iterative process, the derivative of  $f(d)$  is required. Thus,

$$f'(d) = \left( 2 \cdot d \cdot E - \frac{d^2 \cdot E}{V} \cdot \log 0.05 \right) \cdot 0.05^{-\frac{d}{V}}$$

Equation 18    Derivative of Function  $f$

#### **A.1.1**    **ITERATIVE PROCESS**

1. The iteration process is started with an initial guess of the distance,  $d_0$ . Given the typical distances used, a value of 10000 m may be an acceptable starting point.
2. A better approximation is then found with the value:

$$d_{n+1} = d_n - \frac{f(d_n)}{f'(d_n)}$$

Equation 19    Finding a better approximation of the distance

Where:

$d_n$  is the current value for the distance  $d$ .

$d_{n+1}$  is the next approximation of distance  $d$  in the iteration process.

$f(d_n)$  is the value of Equation 17 with the value of  $d_n$ .

$f'(d_n)$  is the value of Equation 18 with the value of  $d_n$ .

3. Repeat step 2 by setting  $d_n = d_{n+1}$  until  $|d_{n+1} - d_n| < \varepsilon$ . The value of  $\varepsilon$  is chosen to be a small value less than the required resolution from the calculation. For example, if 1 m resolution is required, a value of 0.1 m may be acceptable for  $\varepsilon$ .
4. The distance calculated is the final value of  $d_{n+1}$ . This is the luminous range.

The input values should be restricted to specific intervals to ensure that the iteration converges, and does not form a never ending loop.

Practical limitations may be:

Illuminance:  $10^{-7} \text{ lx} \leq E \leq 0.01 \text{ lx}$

Visibility:  $370.4 \text{ m} \leq V \leq 55\,560 \text{ m}$  (0.2 M to 30 M)

Luminous intensity:  $0.1 \text{ cd} \leq I \leq 10\,000\,000 \text{ cd}$

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